Math 2010 Assignment V Prof. Y. Kwong

1/. Given $f(x,y,z) = xe^{y} + y\sin z$, suppose the measurements on x, y and z could have maximum possible errors of ± 0.2, ± 0.6 and $\pm \frac{\pi}{180}$ respectively. Use differentials i.e. $df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$ to estimate the magnitude of the maximum possible error in calculating f from the measured values $x=2$, $y=\ln 3$ and $z=\pi/2$ (Hint: take dx, dy & dz be errors in x, y and z).

2/. Given $f(x,y) = y\sin x$, for any point $(a,b) \in \mathbb{R}^2$, find the total derivative of f, $f'(a,b)$ and verify that it indeed satisfies the definition of total derivative i.e. $f(a+h_1, b+h_2) = f(a,b) + f'(a,b) \cdot \langle h_1, h_2 \rangle + o\left(\sqrt{h_1^2+h_2^2}\right)$.

3/. Suppose you are standing at the point $(-100, -100, 430)$ on a hill whose shape is given by the surface

$$z = 500 - (0.003)x^2 - (0.004)y^2$$

with x, y and z in feet. (a) What is the rate of climb (i.e. the rise in height per unit distance travelled horizontally) if you are heading North east? (b) Find the minimum rate of rise and the direction in which it occurs.

4/. Suppose you have a vector-valued function $\vec{f} \mid \mathbb{R}^n \to \mathbb{R}^m$ i.e. $\vec{f}$ assumes the form $\vec{f}(x_1, \cdots, x_n) = \langle f_1(x_1, \cdots, x_n), \cdots, f_m(x_1, \cdots, x_n) \rangle$, how would you define the total derivative $\vec{f}'(\vec{a})$, $\vec{a} \in \mathbb{R}^n$. Further, conjecture a formula for $\vec{f}'(\vec{a})$.

5/. Find the points on the rotated ellipse $x^2+xy+y^2=3$ that are nearest and are farest away from the origin. (Hint: note that the system of linear equations $\begin{cases} ax+by=0 \\ cx+dy=0 \end{cases}$ has non-trivial equations iff $ad-bc=0$

6/. The plane $x+y+z=1$ intersects with the vertical cylinder $x^2+y^2=1$ in an ellipse. Using the method of Larange Multipliers to find the points on the ellipse that are nearest and farthest from the origin.

7/. A box with its base in the x-y plane has its four upper vertices on the surface $z = 48 - 3x^2 - 4y^2$, find the maximum possible volume of the box.

8/. Prove that $\forall x, y, z \geq 0$, $\sqrt[3]{xyz} \leq \frac{1}{3}(x+y+z)$ using Calculus.